B.Sc. 1st Semester (Hons.) Examination,

November–2014

PHYSICS

Paper–Phy–101

Mathematical Physics–I

Time allowed : 3 hours] *[Maximum marks : 40*

Note : *Attempt **five** questions in all, selecting at least **two** questions from each section. All questions carry equal marks.*

Section–I

1. (a) If $\vec{A} = \hat{i} + a\,\hat{j} + a^2\,\hat{k}$, $\vec{B} = \hat{i} + b\,\hat{j} + b^2\,\hat{k}$

$\vec{C} = \hat{i} + c\,\hat{j} + c^2\,\hat{k}$ are non-co-planar and

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0, \text{ then show that } abc = -1.$$

4

(b) Show that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ iff $\vec{a}$ and $\vec{c}$ are collinear. 4

2. (a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is time. Find the component of velocity and acceleration at $t = 1$ in the direction of $i + j + 3k$. 4

(b) Find a unit vector perpendicular to the plane of the vectors $\vec{a}$ and $\vec{b}$, if $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$. 4

3. (a) Prove that

$$\vec{a}.\nabla\left(\vec{b}.\nabla\frac{1}{r}\right) = \frac{3(\vec{a}.\vec{r})(\vec{b}.\vec{r})}{r^5} - \frac{\vec{a}.\vec{b}}{r^3} \qquad 4$$

(b) Find the angle of intersection at (4, – 3, 2) of spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y + 8z - 4 = 0$ 4

4. State and prove Gauss theorem. 8

Section–II

5. (a) If (r, θ, ϕ) are spherical co-ordinates, show that

$$\nabla\left(\frac{1}{r}\right) = \nabla \times (\cos\theta\, \nabla\phi) \qquad 4$$

(b) Show that in orthogonal co-ordinates

$$\nabla \cdot (A_3 e_3) = \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial w} (A_3 h_1 h_2) \qquad 4$$

6. Discuss the method of Lagrange undetermined multiplier and its application. 8

7. (a) Evaluate $\iint_S \vec{f} \cdot \hat{n}\, dS$, where

$\vec{f} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane 2x + y + 2z = 6 in the first octant. 4

(b) If $\vec{f} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, evaluate $\iiint_V \nabla \times \vec{f}\, dV$, where V is the region bounded by the co-ordinate plane and the plane $2x + 2y + z = 4$. 4

8. Derive Euler-Lagrange equation. Find the shortest curve joining the fixed points. 8